ARRAY RESPONSE KERNELS FOR EEG/MEG IN SINGLE-SHELL ELLIPSOIDAL GEOMETRY

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ABSTRACT

We present analytical forward modeling solutions in the form of array response kernels for electroencephalography (EEG) and magnetoencephalography (MEG) assuming a single-shell ellipsoidal geometry that approximates the anatomy of the head and a dipole current models the source. The structure of our solution facilitates the analysis of the inverse problem by factoring the lead field into a product of the current dipole source and a kernel containing the information corresponding to the head geometry and location of the source and sensors. This factorization allows the inverse problem to be approached as an explicit function of just the location parameters, which reduces the complexity of the estimation solution search. Furthermore, the use of an ellipsoidal geometry is useful for cases when incorporating the anisotropy of the head is important but a better model cannot be defined.

1. INTRODUCTION

Array processing methods have been developed to solve problems related to the localization of brain activity sources using electroencephalography (EEG) and magnetoencephalography (MEG) sensor arrays, and the solutions are useful in neurosciences and clinical applications [1], [2]. Solution to the forward modeling problem in EEG/MEG consists of computing the electric potentials over the scalp and the magnetic field outside the head, respectively, given a current source within the brain. The forward model is necessary for solving the inverse problem (i.e., finding the current distributions using EEG/MEG measurements). Since solving the inverse problem many times involves an iterative solution of the forward problem, it is important to have an efficient form for the analytical and numerical solutions of the forward problem in order to minimize the computational burden [3].

In this paper, we present analytical forward modeling solutions in the form of array response kernels for EEG/MEG assuming a single-shell ellipsoidal geometry that approximates the anatomy of the head and a dipole current models the source. The structure of our solution facilitates the analysis of the inverse problem by decoupling the dipole source signal (linear parameter) from the source location (nonlinear parameter). We factor the lead field into a product of the current dipole source and kernel. This factorization allows the inverse problem to be approached as an explicit function of just the location parameters, which reduces the complexity of the estimation solution search [4].

The use of an ellipsoidal geometry to model the head is useful for cases when incorporating the anisotropy of the head is important but a better model cannot be defined. This is the case, for instance, of fetal MEG studies [5], where the inaccessibility to the fetal head as well as health issues do not permit the use of tomographic techniques to obtain more realistic head models. Furthermore, the ellipsoidal model is useful in MEG studies in adults as it decouples not only the source location from the dipole moment, but also from the sensor location, allowing for further simplification in the computations.

Recently, new expressions for the electric potentials and magnetic fields in single-shell ellipsoidal geometry have been developed [6], [7]. However, these expressions are not suitable for direct use in the inverse neuroelectromagnetic problem. In Section 2, we introduce the original formulations of the forward solutions for both EEG and MEG, while in Section 3 we present the algebraic steps necessary to manipulate those and take them to their factorized forms. In Section 4 we discuss the results and future work.

2. FORWARD MODELING SOLUTIONS

In this section, we present the derivation of the solution to the forward problem of computing the magnetic field outside an ellipsoidal conductor and the electric potential over the surface due to a current dipole source.

2.1. Biot-Savart-Maxwell Solution

We model both the neuroelectric and neuromagnetic phenomena using the quasi-static approximation of Maxwell's equations given that the time-derivatives of the associated electric and magnetic fields are sufficiently small to be ignored [8]. Under this condition, the static electromagnetic field equations can be written as

$$\begin{aligned} \nabla \times \boldsymbol{b}(\boldsymbol{r}) &= \mu_0 \boldsymbol{J}(\boldsymbol{r}), && (1)\\ \nabla \times \boldsymbol{E}(\boldsymbol{r}) &= \boldsymbol{0}, && (2)\\ \nabla \cdot \boldsymbol{b}(\boldsymbol{r}) &= 0, && (3)\\ \nabla \cdot \boldsymbol{E}(\boldsymbol{r}) &= 0, && (4) \end{aligned}$$

where $\boldsymbol{b}$ is the magnetic field, $\boldsymbol{E}$ is the electric field, $\boldsymbol{r} = [r_x, r_y, r_z]^T$ is the observation point, μ_0 is the magnetic permeability (assumed to be the same inside or outside the brain), and $\boldsymbol{J}$ is the current density. Since $\boldsymbol{E}$ is irrotational, it can be represented in terms of the electric potential v as

$$\boldsymbol{E}(\boldsymbol{r}) = -\nabla v(\boldsymbol{r}). \quad (5)$$

The current density can be divided into *passive* and *primary* components. The passive currents $\boldsymbol{J}^{\mathrm{v}}$ are result of the *macroscopic* electric field in the conducting medium, and are described by the following expression

$$\boldsymbol{J}^{\mathrm{v}}(\boldsymbol{r}) = \sigma(\boldsymbol{r})\boldsymbol{E}(\boldsymbol{r}), \quad (6)$$

where σ is the electric conductivity. The primary currents $\boldsymbol{J}^{\mathrm{p}}$ can be considered as the difference of the *impressed* neural current and the *microscopic* passive cellular currents:

$$\boldsymbol{J}^{\mathrm{p}}(\boldsymbol{r}) = \boldsymbol{J}(\boldsymbol{r}) - \boldsymbol{J}^{\mathrm{v}}(\boldsymbol{r}). \quad (7)$$

Under these conditions, the equation that relates $\boldsymbol{b}$ to $\boldsymbol{J}^{\mathrm{p}}$ is the integral form of the Biot-Savart-Maxwell law [9]

$$\boldsymbol{b}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\int_V [\boldsymbol{J}^{\mathrm{p}}(\boldsymbol{r}') - \sigma\nabla v(\boldsymbol{r}')] \times \frac{\boldsymbol{r}-\boldsymbol{r}'}{|\boldsymbol{r}-\boldsymbol{r}'|^3} dV(\boldsymbol{r}'), \quad (8)$$

where $\boldsymbol{r}'$ is the source point, and V indicates the space interior to the volume. Equation (8) is derived under the assumption that our volume is bounded by a single layer with interior homogeneous conductivity σ and exterior conductivity of zero. Similarly, v and $\boldsymbol{J}^{\mathrm{p}}$ are related by

$$v(\boldsymbol{r}) = \frac{1}{4\pi\sigma}\int_V [\boldsymbol{J}^{\mathrm{p}}(\boldsymbol{r}') - \sigma\nabla v(\boldsymbol{r}')] \cdot \frac{\boldsymbol{r}-\boldsymbol{r}'}{|\boldsymbol{r}-\boldsymbol{r}'|^3} dV(\boldsymbol{r}'). \quad (9)$$

Assume that the source is modeled by an equivalent current dipole (ECD), i.e.

$$\boldsymbol{J}^{\mathrm{p}}(\boldsymbol{r}) = \boldsymbol{q}\delta(\boldsymbol{r}-\boldsymbol{r}_0), \quad (10)$$

where $\boldsymbol{q} = [q_x, q_y, q_z]^T$ is the dipole moment, and $\boldsymbol{r}_0 = [r_{0x}, r_{0y}, r_{0z}]^T$ is the source location. The ECD model is a common simplification justified when the source dimensions are relatively small compared with the distances from the source to measurement sensors [10], as it is often true for evoked response and event related experiments. Substituting the ECD model in (8) and (9), and transforming them to surface integrals through simple vector identities, we have the following relationships

$$\begin{aligned} \boldsymbol{b}(\boldsymbol{r}) = {} & \frac{\mu_0}{4\pi}\boldsymbol{q} \times \frac{\boldsymbol{r}-\boldsymbol{r}_0}{|\boldsymbol{r}-\boldsymbol{r}_0|^3} \\ & - \frac{\mu_0\sigma}{4\pi}\int_S v(\boldsymbol{r}')\boldsymbol{u}' \times \frac{\boldsymbol{r}-\boldsymbol{r}'}{|\boldsymbol{r}-\boldsymbol{r}'|^3} dS(\boldsymbol{r}'), \quad (11) \end{aligned}$$

where $\boldsymbol{u}'$ is the outward unit vector normal to the surface S at a point $\boldsymbol{r}'$, and

$$\begin{aligned} v(\boldsymbol{r}) = {} & \frac{1}{4\pi\sigma}\boldsymbol{q} \cdot \frac{\boldsymbol{r}-\boldsymbol{r}_0}{|\boldsymbol{r}-\boldsymbol{r}_0|^3} \\ & - \frac{1}{4\pi}\int_S v(\boldsymbol{r}')\boldsymbol{u}' \cdot \frac{\boldsymbol{r}-\boldsymbol{r}'}{|\boldsymbol{r}-\boldsymbol{r}'|^3} dS(\boldsymbol{r}'). \quad (12) \end{aligned}$$

2.2. Forward Solutions for an Ellipsoidal Volume

Assume that S corresponds to an ellipsoid defined by the following equation

$$\frac{x^2}{\alpha_1^2} + \frac{y^2}{\alpha_2^2} + \frac{z^2}{\alpha_3^2} = 1, \quad (13)$$

where $\alpha_1, \alpha_2, \alpha_3$ are the semiaxes of the ellipsoid. Suppose, without loss of generality, that $+\infty > \alpha_1 > \alpha_2 > \alpha_3 > 0$. Then, equation (13) defines an ellipsoidal system [11] with coordinates (ρ, μ, ν) such that

$$\rho \in \left[\sqrt{\alpha_1^2 - \alpha_3^2}, +\infty\right], \ \mu \in \left[\sqrt{\alpha_1^2 - \alpha_2^2}, \sqrt{\alpha_1^2 - \alpha_3^2}\right],$$

and

$$\nu \in \left[-\sqrt{\alpha_1^2 - \alpha_2^2}, \sqrt{\alpha_1^2 - \alpha_2^2}\right].$$

Using (13) in the evaluation of (11), the solution for the magnetic field in Cartesian coordinates becomes [6]

$$\begin{aligned}\boldsymbol{b}(\boldsymbol{r}) = & \frac{\mu_0}{4\pi}\frac{\mathbb{F}_2^1(\rho_r,\mu_r,\nu_r)}{\gamma_1-\gamma_2}\sum_{i=1}^{3}\frac{\tilde{\boldsymbol{q}}\cdot\boldsymbol{u}_i}{\gamma_1-\alpha_i^2}\boldsymbol{u}_i \\ & -\frac{\mu_0}{4\pi}\frac{\mathbb{F}_2^2(\rho_r,\mu_r,\nu_r)}{\gamma_1-\gamma_2}\sum_{i=1}^{3}\frac{\tilde{\boldsymbol{q}}\cdot\boldsymbol{u}_i}{\gamma_2-\alpha_i^2}\boldsymbol{u}_i \\ & -\frac{15\mu_0}{4\pi}\sum_{\substack{i,j=1\\ i\neq j}}^{3}(\tilde{\boldsymbol{q}}\cdot\boldsymbol{u}_i)(\boldsymbol{r}\cdot\boldsymbol{u}_i)(\boldsymbol{r}\cdot\boldsymbol{u}_j) \\ & \times\mathbb{I}_2^{i+j}(\rho_r)\boldsymbol{u}_j + O(\mathrm{el}_3),\end{aligned} \quad (14)$$

where (ρ_r,μ_r,ν_r) refer to the components of $\boldsymbol{r}$ in the ellipsoidal coordinate system; $\tilde{\boldsymbol{q}} = [\tilde{q}_x,\tilde{q}_y,\tilde{q}_z]^T$ is the dipole moment $\boldsymbol{q}$ modified by the spatial effects of the anisotropy imposed by the ellipsoid; $\mathbb{F}_2^1(\cdot)$ and $\mathbb{F}_2^2(\cdot)$ are the second degree exterior solid ellipsoidal harmonics of orders 1 and 2, respectively; $\mathbb{I}_2^{i+j}(\cdot)$ is the second degree elliptical integral of order $i+j$; $\boldsymbol{u}_i$ is the 3-dimension vector with "1" in the ith position and zero elsewhere; γ_1 and γ_2 are the roots of the quadratic equation $\sum_{i=1}^{3} 1/(\gamma-\alpha_i^2)$; $O(\mathrm{el}_3)$ denotes ellipsoidal terms of degrees greater or equal to three.

Similarly, if we evaluate (11) over (13), then the solution for the electric potential is given by [7]

$$\begin{aligned}v(\boldsymbol{r}) = & \frac{3}{4\pi\sigma\alpha_1\alpha_2\alpha_3}\sum_{i=1}^{3}\frac{(\boldsymbol{q}\cdot\boldsymbol{u}_i)(\boldsymbol{r}\cdot\boldsymbol{u}_i)\mathbb{I}_1^i(\rho_r)}{\mathbb{I}_1^i(\alpha_1)} \\ & -\frac{5}{8\pi\sigma\alpha_1\alpha_2\alpha_3(\gamma_1-\gamma_2)}\sum_{i=1}^{3}(\boldsymbol{q}\cdot\boldsymbol{u}_i)(\boldsymbol{r}_0\cdot\boldsymbol{u}_i) \\ & \times\left[\frac{\mathbb{I}_2^1(\rho_r)\mathbb{E}_2^1(\rho_r,\mu_r,\nu_r)}{\gamma_1(\gamma_1-\alpha_i^2)\mathbb{I}_2^1(\alpha_1)} - \frac{\mathbb{I}_2^2(\rho_r)\mathbb{E}_2^2(\rho_r,\mu_r,\nu_r)}{\gamma_2(\gamma_2-\alpha_i^2)\mathbb{I}_2^2(\alpha_1)}\right] \\ & +\frac{15}{4\pi\sigma\alpha_1\alpha_2\alpha_3}\sum_{\substack{i,j=1\\ i\neq j}}^{3}(\boldsymbol{q}\cdot\boldsymbol{u}_i)(\boldsymbol{r}\cdot\boldsymbol{u}_i)(\boldsymbol{r}\cdot\boldsymbol{u}_j)(\boldsymbol{r}_0\cdot\boldsymbol{u}_j) \\ & \times\frac{\mathbb{I}_2^{i+j}(\rho_r)}{(\alpha_i^2+\alpha_j^2)\mathbb{I}_2^{i+j}(\alpha_1)} + O(\mathrm{el}_3),\end{aligned} \quad (15)$$

where $\mathbb{E}_2^1(\cdot)$ and $\mathbb{E}_2^2(\cdot)$ are the second degree interior solid ellipsoidal harmonics of order 1 and 2, respectively.

Clearly, equations (14) and (15) are not suitable for a numerical solution of the inverse problem in EEG/MEG. Therefore, in the next section we develop the corresponding factorized expressions through a series of algebraic manipulations.

3. ARRAY RESPONSE KERNELS

In this section, we develop novel reformulations to the forward solutions (14) and (15) based on algebraic factorizations. Our goal is then to represent the magnetic field as

$$\boldsymbol{b}(\boldsymbol{r}) = K(\boldsymbol{r},\boldsymbol{r}_0)\boldsymbol{q}, \quad (16)$$

where $K(\boldsymbol{r},\boldsymbol{r}_0)$ is the 3×3 *kernel matrix* for the MEG case, and the electric potential as

$$v(\boldsymbol{r}) = \boldsymbol{k}^T(\boldsymbol{r},\boldsymbol{r}_0)\boldsymbol{q}, \quad (17)$$

where $\boldsymbol{k}(\boldsymbol{r},\boldsymbol{r}_0)$ is the 3×1 *kernel vector* for the case of EEG.

3.1. MEG Kernel Matrix

In order to reach the form of (16), we first need to factorize the modified dipole $\tilde{\boldsymbol{q}}$, originally defined in [6] as

$$\begin{aligned}\tilde{\boldsymbol{q}} = & \frac{\alpha_2^2 q_y r_{0z} - \alpha_3^2 q_z r_{0y}}{\alpha_2^2+\alpha_3^2}\boldsymbol{u}_1 + \frac{\alpha_3^2 q_z r_{0x} - \alpha_1^2 q_x r_{0z}}{\alpha_1^2+\alpha_3^2}\boldsymbol{u}_2 \\ & + \frac{\alpha_1^2 q_x r_{0y} - \alpha_2^2 q_y r_{0x}}{\alpha_1^2+\alpha_2^2}\boldsymbol{u}_3.\end{aligned} \quad (18)$$

We can rewrite this expression in a matrix form by defining $H(\boldsymbol{r}_0)$ as

$$H(\boldsymbol{r}_0) \triangleq \begin{bmatrix} 0 & \frac{\alpha_2^2 r_{0z}}{\alpha_2^2+\alpha_3^2} & \frac{-\alpha_3^2 r_{0y}}{\alpha_2^2+\alpha_3^2} \\ \frac{-\alpha_1^2 r_{0z}}{\alpha_1^2+\alpha_3^2} & 0 & \frac{\alpha_3^2 r_{0x}}{\alpha_1^2+\alpha_3^2} \\ \frac{\alpha_1^2 r_{0y}}{\alpha_1^2+\alpha_2^2} & \frac{-\alpha_2^2 r_{0x}}{\alpha_1^2+\alpha_2^2} & 0 \end{bmatrix}, \quad (19)$$

then

$$\tilde{\boldsymbol{q}} = H(\boldsymbol{r}_0)\boldsymbol{q}. \quad (20)$$

Similarly, we define the support matrices Γ_1, Γ_2, and $\Omega(\boldsymbol{r})$ as

$$\Gamma_l \triangleq \frac{\boldsymbol{u}_1\boldsymbol{u}_1^T}{\gamma_l-\alpha_1^2} + \frac{\boldsymbol{u}_2\boldsymbol{u}_2^T}{\gamma_l-\alpha_2^2} + \frac{\boldsymbol{u}_3\boldsymbol{u}_3^T}{\gamma_l-\alpha_3^2}, \text{ for } l=1,2, \quad (21)$$

$$\Omega(\boldsymbol{r}) \triangleq \begin{bmatrix} 0 & r_x r_y \mathbb{I}_2^3(\rho_r) & r_x r_z \mathbb{I}_2^4(\rho_r) \\ r_x r_y \mathbb{I}_2^3(\rho_r) & 0 & r_y r_z \mathbb{I}_2^5(\rho_r) \\ r_x r_z \mathbb{I}_2^4(\rho_r) & r_y r_z \mathbb{I}_2^5(\rho_r) & 0 \end{bmatrix}. \quad (22)$$

Using (20)–(22) in (14), and discarding the higher order terms, we can express $K(\boldsymbol{r},\boldsymbol{r}_0)$ as [5]

$$\begin{aligned}K(\boldsymbol{r},\boldsymbol{r}_0) = & \frac{\mu_0}{4\pi}\left[\frac{\mathbb{F}_2^1(\rho_r,\mu_r,\nu_r)}{\gamma_1-\gamma_2}\Gamma_1\right. \\ & \left.-\frac{\mathbb{F}_2^2(\rho_r,\mu_r,\nu_r)}{\gamma_1-\gamma_2}\Gamma_2 - 15\Omega(\boldsymbol{r})\right]H(\boldsymbol{r}_0).\end{aligned} \quad (23)$$

We can further simplify (23) by defining $G(\boldsymbol{r})$ as

$$\begin{aligned}G(\boldsymbol{r}) \triangleq & \frac{\mu_0}{4\pi}\left[\frac{\mathbb{F}_2^1(\rho_r,\mu_r,\nu_r)}{\gamma_1-\gamma_2}\Gamma_1\right. \\ & \left.-\frac{\mathbb{F}_2^2(\rho_r,\mu_r,\nu_r)}{\gamma_1-\gamma_2}\Gamma_2 - 15\Omega(\boldsymbol{r})\right],\end{aligned} \quad (24)$$

then $K(\boldsymbol{r},\boldsymbol{r}_0) = G(\boldsymbol{r})H(\boldsymbol{r}_0)$. This result has the advantage of decoupling not only $\boldsymbol{q}$ and $\boldsymbol{r}$, but also $\boldsymbol{r}$ and $\boldsymbol{r}_0$.

3.2. EEG Kernel Vector

In a similar way as in section 3.1, we can define the auxiliary vectors $\boldsymbol{g}(\boldsymbol{r})$, $\boldsymbol{h}_1(\boldsymbol{r}, \boldsymbol{r}_0)$, and $\boldsymbol{h}_2(\boldsymbol{r}, \boldsymbol{r}_0)$ as

$$\boldsymbol{g}(\boldsymbol{r}) \triangleq \left[\sum_{i=1}^{3} \frac{\mathbb{I}_1^i(\rho_r)}{\mathbb{I}_1^i(\alpha_1)} \boldsymbol{u}_i \boldsymbol{u}_i^T\right] \boldsymbol{r}, \tag{25}$$

$$\begin{aligned} \boldsymbol{h}_1(\boldsymbol{r}, \boldsymbol{r}_0) \triangleq \Bigg\{ \sum_{i=1}^{3} \Bigg[& \frac{\mathbb{I}_2^1(\rho_r)\mathbb{E}_2^1(\rho_r, \mu_r, \nu_r)}{\mathbb{I}_2^1(\alpha_1)\gamma_1(\gamma_1 - \alpha_i^2)} \\ & - \frac{\mathbb{I}_2^2(\rho_r)\mathbb{E}_2^2(\rho_r, \mu_r, \nu_r)}{\mathbb{I}_2^2(\alpha_1)\gamma_2(\gamma_2 - \alpha_i^2)} \Bigg] \boldsymbol{u}_i \boldsymbol{u}_i^T \Bigg\} \boldsymbol{r}_0, \end{aligned} \tag{26}$$

$$\boldsymbol{h}_2(\boldsymbol{r}, \boldsymbol{r}_0) \triangleq$$

$$\begin{bmatrix} 0 & \frac{r_x r_y \mathbb{I}_2^3(\rho_r)}{(\alpha_1^2+\alpha_2^2)\mathbb{I}_2^3(\alpha_1)} & \frac{r_x r_z \mathbb{I}_2^4(\rho_r)}{(\alpha_1^2+\alpha_3^2)\mathbb{I}_2^4(\alpha_1)} \\ \frac{r_x r_y \mathbb{I}_2^3(\rho_r)}{(\alpha_1^2+\alpha_2^2)\mathbb{I}_2^3(\alpha_1)} & 0 & \frac{r_y r_z \mathbb{I}_2^5(\rho_r)}{(\alpha_2^2+\alpha_3^2)\mathbb{I}_2^5(\alpha_1)} \\ \frac{r_x r_z \mathbb{I}_2^4(\rho_r)}{(\alpha_1^2+\alpha_3^2)\mathbb{I}_2^4(\alpha_1)} & \frac{r_y r_z \mathbb{I}_2^5(\rho_r)}{(\alpha_2^2+\alpha_3^2)\mathbb{I}_2^5(\alpha_1)} & 0 \end{bmatrix} \boldsymbol{r}_0. \tag{27}$$

Therefore, using (25)–(27) in (15), $\boldsymbol{k}(\boldsymbol{r}, \boldsymbol{r}_0)$ is expressed as

$$\begin{aligned} \boldsymbol{k}(\boldsymbol{r}, \boldsymbol{r}_0) = \frac{1}{4\pi\sigma\alpha_1\alpha_2\alpha_3} \Big[& 3\boldsymbol{g}(\boldsymbol{r}) \\ & - \frac{5}{2}\boldsymbol{h}_1(\boldsymbol{r}, \boldsymbol{r}_0) + 15\boldsymbol{h}_2(\boldsymbol{r}, \boldsymbol{r}_0) \Big]. \end{aligned} \tag{28}$$

In this case, decoupling $\boldsymbol{r}$ and $\boldsymbol{r}_0$ in the EEG kernel is not possible.

4. CONCLUSIONS

We presented analytical solutions to the EEG/MEG forward problem in the form of array response kernels for the case of a sigle-shell ellipsoidal head model. The kernel structure of our forward solutions has the potential of facilitating the solution of the inverse problem as it provides with an algebraic representation suitable for numerical implementations. The simplification is greater in the case of MEG where all the parameters are decoupled from each other. Further work in this area includes the extension of our expressions to the case of multi-shell ellipsoidal head model, as well as extending these models to line- and surface-dipole models.

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