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MEG Source Estimation in the Presence of Low-Rank Interference Using Cross-Spectral Metrics

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Abstract—**We estimate a source current dipole at a known location in the presence of low-rank interference using magnetoencephalography (MEG). We present a space-time processor for MEG data based on the generalized sidelobe canceler (GSC). We extend the classical vector beamformer to a matrix structure without making any assumptions on the rank of the covariance matrix of noise and interference, or constraint matrices. Furthermore, we define the cross-spectral metrics (CSM) in their most general form. The CSM method is known to approximate the performance of the matched filter for the case of unknown covariance matrix. In our case, the CSM also allows to reduce the complexity of the filtering problem without significant loss of performance in the signal-to-interference-plus-noise ratio (SINR). Our results show that good estimates of the dipole sources can be achieved by only using a few eigenvalues, namely, those corresponding to the largest CSM.**

Keywords—**Beamforming, source estimation, magnetoencephalography, cross-spectral metric, generalized sidelobe canceler, low-rank interference**

I. Introduction

Sensor array processing techniques have been used to solve problems related to space-time processing of neuroelectric and neuromagnetic signals. These problems include the localization of brain activity sources using electroencephalography (EEG) and magnetoencephalography (MEG) sensor arrays [1], as well as source signal reconstruction and interference cancellation [2]. In this paper, we consider the problem of source signal estimation using MEG when the interference is low-rank. Interference arises in many practical signal processing applications, and accounting for its low-rank property may reduce the number of interference parameters by an order of magnitude [3].

The theory of beamforming processing has been developed for radar or sonar applications where the response of the array can be expressed in a complex vector form, commonly referred as the *steering vector* [4]. However, in the case of bioelectromagnetic measurements, the response of the array to a dipole source corresponds to a collection of space-time processors usually arranged in matrix form (spatio-temporal matrix). Therefore, our goal in this paper is to extend the classical vector processor to a more general matrix form able to handle this specific case of array processing.

Current methods for source reconstruction and interference cancellation often rely on eigenvalue decomposition and principal component (PC) selection under the assumption that the interference is low rank and its spatial eigenvectors are different from the lead field of a brain source of interest [5]. In this case, the eigenvectors of the data covariance matrix related to such interference are very different from any of the lead field vectors in the source subspace. However, this assumption may not hold for some interference with biological origins, such as eye blinking, cardiac sources, or even background brain activity (e.g. α waves) [6]-[8]. Then, problems may arise when a signal eigenvalue is mistakenly swapped with the noise subspace eigenvalues. Furthermore, the majority of current methods use algorithms based on diagonal loading (i.e., adding a small positive quantity to diagonal elements of the covariance matrix of interference and noise to make it invertible [9]), which results in suboptimal solutions.

Here, we propose a different approach to low-rank interference mitigation: instead of "forcing" the covariance matrix to be nonsingular, we assume that it has a low rank and use optimal filters [10] for this scenario. Specifically, our technique uses a reduced-rank generalized sidelobe canceler (GSC) [11], where the reduction of the rank is based in the cross-spectral metrics (CSM) [12]. The GSC is a useful implementation of the linearly constrained minimum variance (LCMV) beamformer that allows to modify the principal component approach to include signal dependency, and provides insight to the rank-reduction and dimensionality reduction problems [13]. In order to reduce the complexity of the GSC, we propose to use the CSM method for rank-reduction as it has been demonstrated that this metric allows the dimensionality of the processor to be reduced below the dimension of the noise subspace eigenstructure without significant loss of performance in the signal-to-interference-plus-noise ratio (SINR), and that the cross-spectral subspace reduced-dimension processor can outperform the full-dimension processor when the noise covariance is unknown, closely approximating the performance of the matched filter [14].

In this paper, we extend the GSC and the CSM from the classical steering vector expressions to a matrix form, and apply the generalized rank-reduced processor to estimate dipole source components with MEG data. In Section II, we define our MEG measurement model under the presence of additive Gaussian noise and pose the LCMV problem for interference cancellation. In Section III, we define the structure of the GSC and derive a general expression for the SINR. This expression is used in Section IV to derive the CSM, where we also define the reduced-rank GSC. The applicability of this processor to a typical MEG array system is shown through numerical simulations in Section V. Finally, in Section VI, we discuss the results, limitations, and further work.

II. Measurement Model

Consider the case of measuring the magnetic field outside the head produced by L dipole sources with moments $\boldsymbol{q}_l = [q_{lx}, q_{ly}, q_{lz}]^T$, for $l = 1, \ldots, L$. Assume that the sources change in time and remain in the same position $\boldsymbol{\theta}$ during the measurements period. This assumption holds in practice for evoked response and event-related experiments. Then, MEG data is collected by an m-element sensor array at time samples $t = 1, 2, \ldots, N$. Let Y_k denote the $m \times N$ spatio-temporal data matrix obtained from the MEG sensors at the kth trial, for $k = 1, 2, \ldots, K$, where K is the number of independent experiments. Then, the measurement model is given by

$$Y_k = A(\boldsymbol{\theta}) Q_k + E_k, \tag{1}$$

where $A(\boldsymbol{\theta})$ is the $m \times 3L$ array response matrix, $Q_k = [{\boldsymbol{q}_1}^T, \ldots, {\boldsymbol{q}_L}^T]^T$ at the kth trial, and E_k is the noise and interference matrix (considered to be arbitrary but constant between trials). Assume that the measurements are taken in the presence of zero mean Gaussian noise uncorrelated between samples. Using a vector representation, we can rewrite equation (1) as

$$\mathrm{vec}(Y_k) = \boldsymbol{y}_k = [I_N \otimes A(\boldsymbol{\theta})]\mathrm{vec}(Q_k) + \mathrm{vec}(E_k). \tag{2}$$

Define $\boldsymbol{e}_k = \mathrm{vec}(E_k)$, we can then express the covariance matrix of the noise and interference as $R = \mathrm{E}[\boldsymbol{e}_k \boldsymbol{e}_k^T]$. Now, for the case of unknown R, we can obtain a consistent estimate of this covariance matrix as

$$\widehat{R} = \left(\frac{1}{K} \sum_{k=1}^{K} \boldsymbol{y}_k \boldsymbol{y}_k^T \right) - \bar{\boldsymbol{y}}\, \bar{\boldsymbol{y}}^T \text{ where } \bar{\boldsymbol{y}} = \frac{1}{K} \sum_{k=1}^{K} \boldsymbol{y}_k. \tag{3}$$

Our goal is to estimate the dipole components for a given location $\boldsymbol{\theta}$. Define $C = I_N \otimes A(\boldsymbol{\theta})$, and let F be a desired matrix response. Then, we can pose the following LCMV filtering problem

$$\widehat{W} = \min_{W} \; W^T \widehat{R} W \quad \text{subject to} \quad C^T W = F. \tag{4}$$

This equation is an extension to the conventional vector array processor defined in [16].

III. Generalized Sidelobe Canceler

The generalized sidelobe canceler allows us to reformulate the structure of the constrained beamformers in an unconstrained structure with the standard form of a Wiener filter [4]. The GSC is also important as it allows us to easily derive the CSM.

The structure of the GSC is shown in Fig. 1. Here, W_0

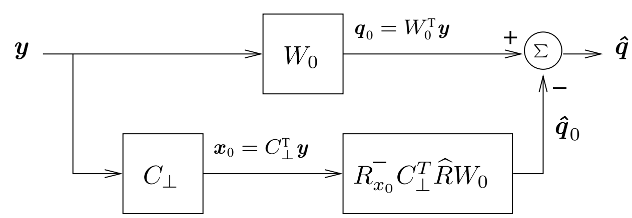

Fig. 1. Structure of the generalized sidelobe canceler.

corresponds to the homogeneous solution of the beamformer (the one that satisfy the *pass* constraints), $\boldsymbol{q}_0$ is the main-beam response, $C_\perp$ is an $mN \times mN - \mathrm{rank}(C)$ blocking matrix whose columns are orthonormal to C, $\boldsymbol{x}_0$ is the blocking response, and $R_{x_0} = \mathrm{E}[\boldsymbol{x}_0 \boldsymbol{x}_0^T] = C_\perp^T \widehat{R} C_\perp$. Note that the term $W_q = R_{x_0}^{-} C_\perp^T \widehat{R} W_0 = R_{x_0}^{-} R_{x_0 q_0}$ corresponds to the Wiener-Hopf solution, where $R_{x_0 q_0} = \mathrm{E}[x_0 q_0^T]$, and $(\cdot)^{-}$ denotes the generalized inverse. In our case, the Wiener solution is slightly more general as we don't make any assumption on the rank of the matrices. Therefore, we can handle the case where $\widehat{R}$ is singular, which will happen for example, if K is small compared with mN. We can also allow the case of low-rank C, which may happen when oversampled point constraints are used.

A. Signal-to-interference-plus-noise Ratio

Consider now a matrix operator $B = [W_0, C_\perp]^T$ transforming the data prior the Wiener filter, i.e. $\tilde{\boldsymbol{y}} = B\boldsymbol{y} = [\boldsymbol{q}_0^T, \boldsymbol{x}_0^T]^T$. Using B, we can convert our GSC to an equivalent structure corresponding to the Wiener filter. This will allow us to easily compute the SINR of the GSC.

Let

$$U = BW_0 = \begin{bmatrix} W_0^T \\ C_\perp^T \end{bmatrix} W_0 = \begin{bmatrix} W_0^T W_0 \\ 0 \end{bmatrix}. \tag{5}$$

In the above expression we used the fact that W_0 depends on C. In our specific case, we use $W_0 = C(C^T C)^{-} F$, where $F = I$, which makes the GSC equivalent to the LCMV beamformer [13]. Note that, due to the orthonormality between B and C, the lower block of the matrix U is canceled.

Let $\mathrm{SINR} = \mathrm{tr}\left\{ U^T R_{\tilde{y}}^{-1} U \right\}$, where $R_{\tilde{y}} = \mathrm{E}[\tilde{\boldsymbol{y}} \tilde{\boldsymbol{y}}^T]$ and $\mathrm{tr}\{\cdot\}$ is the trace operator. Therefore,

$$\mathrm{SINR} = \mathrm{tr} \left\{ U^T \begin{bmatrix} W_0^T R W_0 & R_{x_0 q_0}^T \\ R_{x_0 q_0} & R_{x_0} \end{bmatrix}^{-1} U \right\}. \tag{6}$$

Defining the power of the conventional beamformer output as $P_0 = W_0^T \widehat{R} W_0$, and using the partitioned matrix lemma, we can rewrite the SINR as

$$\mathrm{SINR} = \mathrm{tr}\left\{ W_0^T W_0 \left[P_0 - R_{x_0 q_0}^T R_{x_0}^{-} R_{x_0 q_0} \right]^{-1} \right\}. \quad (7)$$

IV. Cross-Spectral Metric

To compute the CSM, we need to express the SINR in (7) in terms of the eigenvalues and eigenvectors of R_{x_0}. However, we need to define a specific generalized inverse to be able to define the eigendecomposition. Then, we use the Moore-Penrose inverse, denoted as $(\cdot)^+$, and rewrite the SINR as

$$\mathrm{SINR} = \mathrm{tr}\left\{ W_0^T W_0 \left[P_0 - R_{x_0 q_0}^T R_{x_0}^{+} R_{x_0 q_0} \right]^{-1} \right\}. \quad (8)$$

The spectral decomposition of $R_{x_0}^{+}$ is given by

$$R_{x_0}^{+} = V D^{+} V^T \qquad \text{with} \qquad V^T R_{x_0} V = D. \quad (9)$$

In the above expression, D is a diagonal matrix whose first n diagonal elements $\lambda_1, \lambda_2, \ldots, \lambda_n$, are the eigenvalues of R_{x_0} in decreasing order, and $V = [\boldsymbol{v}_1 \; \boldsymbol{v}_2 \; \cdots \; \boldsymbol{v}_n]$, where $\boldsymbol{v}_i, \; i = 1, 2, \ldots, n$, are the orthonormal eigenvectors of R_{x_0} corresponding to $\lambda_1, \lambda_2, \ldots, \lambda_n$. The Moore-Penrose inverse of D is defined as D^{+} where

$$\lambda_i^{+} = \left\{ \begin{array}{ll} 1/\lambda_i & \text{if } \lambda_i \neq 0 \\ 0 & \text{if } \lambda_i = 0 \end{array} \right. . \quad (10)$$

Finally, the eigenvalue decomposition of $R_{x_0}^{+}$ is given by

$$R_{x_0}^{+} = V D^{+} V^T = \sum_{i=1}^{n} \lambda_i^{+} \boldsymbol{v}_i \boldsymbol{v}_i^T. \quad (11)$$

Substituting eqn. (11) in (8), we have

$$\mathrm{SINR} = \mathrm{tr}\left\{ W_0^T W_0 \left[P_0 - \sum_{i=1}^{n} \lambda_i^{+} \boldsymbol{r}_i \boldsymbol{r}_i^T \right]^{-1} \right\}, \quad (12)$$

where $\boldsymbol{r}_i = R_{x_0 q_0}^T \boldsymbol{v}_i$.

Then, the quantity $\varGamma_i = \lambda_i^{+} \boldsymbol{r}_i \boldsymbol{r}_i^T$ is our *matrix* CSM (we will denote it as mCSM). We can also define a *scalar* CSM (sCSM) as

$$\gamma_i = \lambda_i^{+} \mathrm{tr}\left\{ \boldsymbol{r}_i \boldsymbol{r}_i^T \right\}. \quad (13)$$

A. Reduced-rank GSC

An approximate expression for W_q can be obtained using the decomposition in (11):

$$\widetilde{W}_q = \widetilde{R}_{x_0}^{+} R_{x_0 q_0} = \sum_{j} \frac{\boldsymbol{v}_j \boldsymbol{r}_j^T}{\lambda_j}, \quad (14)$$

where the summation index j contains the values of i corresponding to the J largest values of γ_i in descending order. Note that the largest γ_i not necessarily correspond to the largest eigenvalues. However, by basing our selection on the J largest sCSM, we obtain a more efficient reduced-rank processor (greater SINR for a given rank $J \leq N - 1$) than the PC method [15].

V. Numerical Examples

We examine the performance of a reduced-rank GSC using simulated MEG data. We used an array of $m = 37$ electrodes located on a sphere of radius 10.5 cm with a single sensor at the top position and 3 rings at elevation angles of $\pi/12$, $\pi/6$, and $\pi/4$ rad, containing, respectively 6, 12, and 18 sensors equally spaced in the azimuthal direction.

To simulate the sources, we used two dipoles located at a distance of 1.34 cm. The dipole source components are defined as $\boldsymbol{q}_1 = [0.7\rho_1, 0.7\rho_2, 0]^T$ and $\boldsymbol{q}_2 = \rho_3[0.7, 0, 0.5]^T$, where the magnitudes ρ_1, ρ_2, and ρ_3 are allowed to change in time according to

$$\rho_1(t) = 10e^{-(t-100)^2/11^2} - 5e^{-(t-80)^2/17^2}, \quad (15.a)$$

$$\rho_2(t) = 5e^{-(t-80)^2/8^2} - 10e^{-(t-100)^2/11^2}, \quad (15.b)$$

$$\rho_3(t) = 10\sin(0.07\pi t), \quad (15.c)$$

all with units of [nA·m], and t in milliseconds. Similar models has been used in previous research (see e.g. [17]-[18]) as they approximate a typical evoked response. Then, we sampled these signals at a rate of 2 ms thus obtaining $N = 100$ samples for our computer simulations. Note that $\boldsymbol{q}_2$ has harmonic sinusoidal waveforms and acts as an interferer source.

To generate the measurements, we used the forward solution of the MEG spherical radial field described in [19]. Then, we added uncorrelated (in time and space) random noise, distributed as $\mathcal{N}(0, \sigma^2)$ with $\sigma = 35$ fT. Finally, we repeated this process with independent noise realizations to obtain $K = 100$ trials.

Under these conditions, we applied the reduced-rank GSC to estimate the dipole source components. In a first step, we computed the sCSM and chose those with the largest magnitudes. The results showed that, for our specific problem, the largest sCSM corresponds to the eigenvalue located in the 36th position.

Now, using the expression (14) with $j = 36$, we approximated the value of W_q using only one eigenvalue-eigenvector, i.e., the one with the value γ_{36}, which is the largest among all the values of γ_i. With this approximation, we obtained the estimates of the dipole source components. The tangential magnitudes (since MEG is radially silent) of the dipole sources, as well as their estimates, are shown in Fig. 2. Clearly, the approximation based in the largest CSM provides with a good estimate of the sources.

VI. Concluding Remarks

We applied a reduced-rank GSC for the case of estimating the dipole source components using MEG data. The GSC is

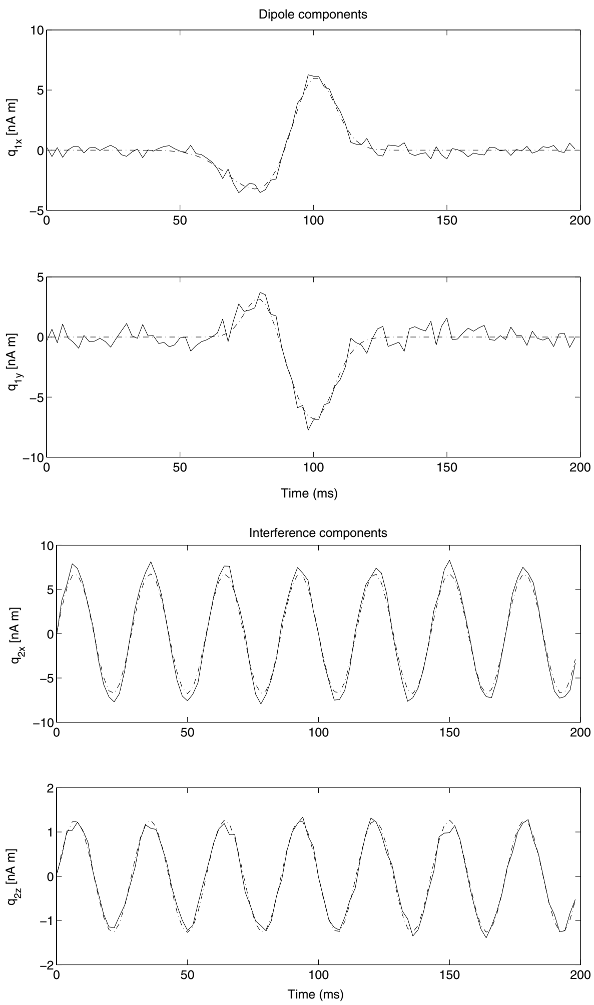

Fig. 2. Tangential magnitudes of the dipole moment and interference source components. The original moments are shown in dash-dotted line and the estimates in solid line.

a useful implementation of the LCMV filtering problem that includes information about signal dependency and provides insight to the rank-reduction problem. Our rank-reduction procedure was based on the CSM, which we defined in a general matrix form. The CSM technique allows us to reduce the subspace dimension of our processor while approximating the performance of the matched filter. In our particular case, this rank-reduced processor showed a good performance in estimating the source components in the presence of interference. However, the CSM method is still limited to the eigen-basis arising from R_{x_0}, which is not directly influenced by the spatio-temporal matrix. Then, future work should look to alternative ways to remedy this shortcoming. Other areas of future investigation include continued research into the performance of different low-rank beamformers using the CSM as benchmark, as well as extensions and modifications with greater robustness to mismatches in the spatio-temporal matrix.

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References

[1] B. D. van Veen, W. van Drongelen, M. Yuchtman, and A. Suzuki, "Localization of brain electrical activity via linearly constrained minimum variance spatial filtering," *IEEE Trans. Biomed. Eng.*, vol.44, no.9, pp. 867-880, Sept. 1997.

[2] K. Sekihara, S. S. Nagarajan, D. Poeppel, A. Marantz, and Y. Miyashita, "Reconstructing spatio-temporal activities of neural sources using an MEG vector beamformer technique," *IEEE Trans. Biomed. Eng.*, vol.48, no.7, pp. 760-771, July 2001.

[3] J. R. Guerci, J. S. Goldstein and I. S. Reed, "Optimal and adaptive reduced-rank STAP," *IEEE Trans. Aerosp. Electron. Syst.*, vol.36, no.2, pp. 647-663, Apr. 2000.

[4] J. R. Guerci, *Space-time adaptive processing techniques for radar*. Artech House, Inc., Norwood, MA, 2003.

[5] K. Sekihara, S. S. Nagarajan, D. Poeppel, A. Marantz, and Y. Miyashita, "Application of an MEG eigenspace beamformer to reconstructing spatio-temporal activities of neural sources," *Human Brain Mapping*, vol.15, no.4, pp. 199-215, 2002.

[6] A. Antervo, R. Hari, T. Katila, T. Ryhänen, and M. Seppänen, "Magnetic fields produced by eye blinking," *Electroenceph. Clin. Neurophysiol.*, vol.61, no.4, pp. 247-253, Oct. 1985.

[7] V. Jousmäki and R. Hari, "Cardiac artifacts in magnetoencephalogram," *J. Clin. Neurophysiol.*, vol.13, no.2, pp. 172-176, Mar. 1996.

[8] J. Knuutila and M. S. Hämäläinen, "Characterization of brain noise using a high sensitivity 7-channel magnetometer," in *Biomagnetism '87*, Tokio Univ. Press, 1988, pp. 186-189.

[9] B. D. Carlson, "Covariance matrix estimation errors and diagonal loading in adaptive arrays," *IEEE Trans. Signal Processing*, vol.24, no.4, pp. 397-401, July 1988.

[10] A. Dogandžić, "Minimum variance beamforming in low-rank interference," in *Proc. 36th Asilomar Conf. Signals, Syst. Comput.*, Pacific Grove, CA, Nov. 2002, pp. 1293-1297.

[11] B. D. van Veen and K. M. Buckeley, "Beamforming: a versatile approach to spatial filtering," *IEEE Signal Processing Mag.*, vol.5, no.2, pp. 4-24, Apr. 1988.

[12] J. S. Goldstein and I. S. Reed, "Subspace selection for partially adaptive sensor array processing," *IEEE Trans. Aerosp. Electron. Syst.*, vol.33, no.2, pp. 539-544, Apr. 1997.

[13] H. L. van Trees, *Optimum array processing: Part IV of Detection, Estimation, and Modulation Theory*. Wiley-Interscience, NY, 2002.

[14] J. S. Goldstein and I. S. Reed, "Theory of partially adaptive radar," *IEEE Trans. Aerosp. Electron. Syst.*, vol.33, no.4, pp. 1309-1325, Oct. 1997.

[15] J. S. Goldstein and I. S. Reed, "Reduced rank adaptive filtering," *IEEE Trans. Signal Processing*, vol.45, no.2, pp. 492-496, Feb. 1997.

[16] O. L. Frost, "An algorithm for linearly constrained adaptive array processing," *Proc. IEEE*, vol.60, pp. 926-935, Aug. 1972.

[17] D. Gutiérrez, A. Nehorai, and C. Muravchik, "Estimating Brain Conductivities and Dipole Source Signals with EEG Arrays," *IEEE Trans. Biomed. Eng.*, in press.

[18] A. Dogandžić and A. Nehorai, "Estimating evoked dipole responses in unknown spatially correlated noise with EEG/MEG arrays," *IEEE Trans. Signal Processing*, vol.SP-48, no.1, pp.3-25, 2000.

[19] J. C. Mosher, R. Leahy, and P. S. Lewis, "EEG and MEG: Forward solutions for inverse methods," *IEEE Trans. Biomed. Eng.*, vol.46, no.3, pp. 245-259, Mar. 1999.